The Bohr Hydrogen Atom as Viewed in the Planck Vacuum Theory

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This paper examines the Bohr hydrogen-atom model from the perspective of the Planck vacuum (PV) theory. The nonrelativistic calculations significantly expand the Bohr concept concerning the quantized angular momentum, the hydrogen energy levels $E_n$, the orbit radii $r_n$, and the hydrogen Rydberg constant $R_H$. Calculations show: that the Bohr quantization of the angular momentum is directly related to the electron/PV coupling force; and that the ratios $E_n/r_n$ are proportional to the n-ratio from the Schwarzschild line element for the Einstein field equations.


1 Introduction

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G} = \left(\frac{m_* c^2}{r_*}\right) = \frac{m_c^2 G}{r_*^2} = \frac{\hbar^2}{r_*^2} \quad (1)$$

where the ratio $c^4/G$ is the curvature superforce that appears in the Einstein field equations. $G$ is Newton’s gravitational constant, $c$ is the speed of light, $m_*$ and $r_*$ are the Planck mass and length respectively [4, p.1234], and $e_*$ is the massless bare charge. The fine structure constant is given by the ratio $\alpha = e^2/e_*^2$, where ($-e$) is the observed electronic charge.

The two particle/PV coupling forces

$$F_e(r) = \frac{e^2}{r^2} - \frac{mc^2}{r} \quad \text{and} \quad F_*(r) = \frac{e_*^2}{r^2} - \frac{m_* c^2}{r} \quad (2)$$

the electron core ($-e_*, m_*$) and the Planck particle core ($e_*, m_*$) exert on the invisible PV state; along with their coupling constants

$$F_e(r_e) = 0 \quad \text{and} \quad F_*(r_*) = 0 \quad (3)$$

and the resulting Compton radii

$$r_e = \frac{e^2}{mc^2} \quad \text{and} \quad r_* = \frac{e_*^2}{m_* c^2} \quad (4)$$

lead to the important string of Compton relations

$$r_e mc^2 = r_* m_* c^2 = e_*^2 \quad (= \hbar c) \quad (5)$$

for the electron and Planck-particle cores, where $\hbar$ is the reduced Planck constant. To reiterate, the equations in (2) represent the forces the free electron and Planck-particle cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores [5].

The Lorentz invariance of the coupling constants in (3) leads to the energy and momentum operators of the quantum theory [5].

The various PV parameters in the previous paragraphs are used in Section 3 to unravel the Bohr-atom results in Section 2 from the single Planck constant $\hbar$ contained therein, leading to a deeper understanding of the Bohr atom defined below.

2 Bohr Atom

The Bohr atom [6] [7, p.73] consists of a massive point-electron ($-e, m_e = (-\alpha^{1/2} e_*, m_*$) and a massive point-proton ($e, M = (\alpha^{1/2} e_*, M_*$) circling their common center of mass. Their masses and circling radii are related by the equation $mr = MR$, where $r$ and $R$ in this equation represent these radii. The $r$ here IS NOT the same as the coordinate radius $r$ in equations (2).

The quantized angular momentum assumed by Bohr is given by [7, p.73]

$$(mr^2 + MR^2) = mr^2 \omega = mr^2 (1 + m/M) = n\hbar \quad (6)$$

in terms of the Planck constant $\hbar$. The centripetal force driving the electron toward the center of mass is given by

$$m\omega^2 r = \frac{e^2}{(r + R)^2} = \frac{e^2}{r^2 (1 + m/M)^2} \quad (7)$$

and the total energy $T + V$ by (using (7))

$$E_n = \frac{mr^2 \omega^2}{2} + \frac{MR^2 \omega^2}{2} - \frac{e^2}{r + R} = - \frac{mr^2 \omega^2}{2} (1 + m/M) \quad (8)$$
where $\omega$ is the angular frequency of the electron and proton about their common center of mass.

Each of the equations (6)–(8) may be solved for $\omega^2 r^4$:

$$\omega^2 r^4 = \frac{n^2 \hbar^2}{m^2 (1 + m/M)^2} = A$$
$$= \frac{e^2 r}{m(1 + m/M)^2} = rB$$
$$= -\frac{2E_n r^2}{m(1 + m/M)} = r^2 (-E_n)C. \quad (9)$$

Eliminating $r$ from the right sides of the above equalities then leads to the energy spectrum ($n = 1, 2, 3, \ldots$)

$$E_n = \frac{B^2}{AC} = \frac{mc^4}{2n^2 \hbar^2 (1 + m/M)} \quad (10)$$

and the electron-orbit radii

$$r_n = \frac{A}{B} = \frac{n^2 \hbar^2}{mc^2}. \quad (11)$$

The wavenumber for the radiation between the energy states $n'$ and $n$ is given by

$$\nu = \frac{E_n - E_{n'}}{2\pi c\hbar} = \frac{1}{\nu} \left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \quad (12)$$

where the Rydberg constant is

$$R_H = \frac{me^4}{4\pi \hbar^3 (1 + m/M)}. \quad (13)$$

These results all depend upon the single constant $\hbar$; so their fundamental physical meaning is obscured. The next section remedies this shortcoming.

3 Planck Vacuum Perspective

This section replaces the single constant $\hbar$ of the previous section by the constants from the PV theory given in the Introduction. It is left for the reader to fill-in the simple steps. The new results due to the PV theory pre-multiply the bracketed quantities in the following equations.

The new angular momentum from (6) becomes

$$(mr^2 + MR^2)\omega = mr^2 \omega (1 + m/M)$$

$$= \sqrt{n\hbar} = \frac{e^2}{c}[n] = r_mc[n]. \quad (14)$$

The squared charge $e^2 = (-e_s)(-e_s)$ in the first equation of (2) represents the numerator of the Coulomb force the electron (the first charge) exerts on the separate Planck particles (the second charge) of the PV state. As such, its appearance in (14) is a manifestation of that coupling force. The $r_mc$ in (14) comes from the Compton relations in (4). Thus, from the first equation in (3), the quantized angular momentum is restricted to those values ($r_mc$) where the electron coupling force $F_e(r_c)$ acting on the vacuum state vanishes, and there are a denumerable infinity ($n = 1, 2, 3, \ldots$) of such values. For $r > r_c$ the electron force in (2) is attractive and for $r < r_c$ it is repulsive.

The new energy levels are given by

$$E_n = -mc^2 \alpha^2 \left[\frac{1}{2n^2(1 + m/M)}\right] \quad (15)$$

where $mc^2$ is the mass energy of the electron and $\alpha$ is the fine structure constant.

The new Bohr electron orbits are

$$r_n = \frac{r_c}{\alpha}[n^2] = \frac{e^2}{\alpha mc^2}[n^2] \quad (16)$$

where $r_c = (e^2/mc^2)$ is the electron Compton radius. This result yields the correct value for $r_1$ with $e_s$ in cgs-esu units [7, p.722].

The energy/orbit ratios are

$$\frac{E_n}{r_n} = -\frac{mc^2}{r_c\alpha^3} \left[\frac{1}{2n^4(1 + m/M)}\right] \quad (17)$$
$$= \frac{m_c c^2}{r_s n_{rc}\alpha^3} \left[\frac{1}{2n^4(1 + m/M)}\right] \quad (18)$$

where

$$n_{rc} \equiv \frac{mc^2/r_c}{m_c c^2/r_s} \quad (19)$$

is the $n$-ratio that appears in the Schwarzschild line element for the Einstein field equations [8].

The new hydrogen-atom Rydberg constant is given by

$$R_H = \frac{\alpha^2}{r_c} \left[\frac{1}{4\pi(1 + m/M)}\right]. \quad (20)$$

This result yields the correct value for the Rydberg constant in cgs-esu units [7, p.723].

4 Conclusions and Comments

It is noted that all of the pre-multipliers in equations (14)–(18) and (20) are PV parameters. Thus the PV theory represents a significant addition to the previous Bohr-atom analysis.

Since the angular momentum has been quantized, the classical variables that are part of the analysis take on new meanings. For example, the angular frequency becomes [for convenience $(1 + m/M) = 1$]

$$\omega = \frac{r_mc[n]}{mr^2} = \frac{r_ec[n]}{r_c^2[n^3]/\alpha^2} = \frac{\alpha c}{r_c} \left[\frac{1}{n^3}\right] \quad (21)$$
where, from (23), \( \alpha c \) is the velocity of the electron in the first \((n = 1)\) Bohr orbit around the center of mass.

Using (21) for \( \omega \), the centripetal acceleration becomes

\[
\omega^2 r = \alpha^4 \frac{e^2}{r_c^2} \left[ \frac{1}{n^6} \right] \frac{r_c}{\alpha} \left[ \frac{1}{n^2} \right] = \alpha \frac{\alpha^2 e^2}{r_c^2} \left[ \frac{1}{n^4} \right] \tag{22}
\]

and the orbit velocity of the electron becomes

\[
v = \omega r = \alpha^2 \frac{c}{r_c} \left[ \frac{1}{n^3} \right] \frac{r_c}{\alpha} \left[ \frac{1}{n^2} \right] = \alpha c \left[ \frac{1}{n} \right]. \tag{23}
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References


