The Dirac Proton and its Structure

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This paper argues that the proton, like the Dirac electron, is a massive "point charge" coupled to the degenerate Planck vacuum state. These two assumptions lead to a definitive proton and antiproton structure. Results show that it is the stressed vacuum state centered on the proton core that leads to an apparent spread of the proton charge. When the energy of an incident electron probing the proton structure reaches 2.95 GeV, the relativistic de Broglie radius (0.067 fm) of the electron equals the radius of the proton electron-proton overlap sphere, and the electron can begin to penetrate the apparent spread. This is the point where the separation between the proton and electron is $0.067 + 0.067 = 0.134$ fm. If this separation is mistakenly attributed solely to the proton, then the resulting Compton wavelength is $2\pi \times 0.134 = 0.842$ fm.

1 Introduction

There is currently a significant discrepancy between the charge and Zemach radii of the proton as measured in the proton-electron scattering, and the muonic-hydrogen experiments [1]. These two experiments place the effective charge radius at 0.879 fm (or 0.875 fm), and 0.84087 fm respectfully. "This discrepancy has triggered a lively discussion addressing the accuracy of the measurements, bound-state QED, proton structure, the Rydberg constant, and possibilities of new physics." It is hoped that the following discussion of the proton structure casts some light on what may be proton (and electron) modeling errors.

In the Planck vacuum (PV) theory [2] [3] [4] [5] the proton and antiproton consist of, respectively, the massive "point charge" cores (e_*, m_p) and $(-e_*, m_p)$ that are coupled to the PV via their coupling forces

$$F(r) = \mp\left(\frac{e_*^2}{r^2} - \frac{m_p c^2}{r}\right) \tag{1}$$

where e_* (5.62×10^{-9} esu) is the massless bare charge and m_p is the proton rest mass. The upper and lower signs in (1) refer to the proton and antiproton cores respectively. [Most of the calculations in this paper take place in the proton or electron rest frames and are meant to be of a heuristic nature.]

The response of the PV to these coupling forces is the two Dirac equations

$$\mp e_*^2\left(i\frac{\partial}{\partial ct} + \boldsymbol{\alpha}\cdot i\nabla\right)\psi = \mp m_p c^2 \beta\psi \tag{2}$$

which are identical after the $\mp$ signs are removed. Dividing through by $\mp m_p c^2$ leads to

$$r_p\left(i\frac{\partial}{\partial ct} + \boldsymbol{\alpha}\cdot i\nabla\right)\psi = \beta\psi \tag{3}$$

for either the proton or antiproton, where $r_p = e_*^2/m_p c^2$ is the Compton radius at which (1) vanishes. Equations (1)-(3) imply that what can be said for the proton, can also be said for the antiproton, and visa versa.

The proton-PV potential energy associated with the coupling force (1) is defined as

$$V(r) = -\int_{r_p}^{r} |F(r)| dr \tag{4}$$

resulting in (using (6))

$$V(r)/m_p c^2 = r_p/r - 1 - \ln(r_p/r) \tag{5}$$

for $r \le r_p$, where $V(r_p) = 0$. For $r > r_p$ the PV is compressed by the coupling force and is not of interest in the following. The potential increases as the proton core is approached, making the negative energy vacuum susceptible to free-space (where the core resides) perturbations. It is seen below that the proton electron-proton (e-p) and proton-proton (p-p) overlap radii (see Appendix A) are important markers for the apparent spread in the charge of the proton core.

2 Proton Structure

The PV state for the electron, proton, and their antiparticles is characterized in part by the three Compton relations

$$r_e m_e c^2 = r_p m_p c^2 = r_* m_* c^2 = e_*^2 \quad (= c\hbar) \tag{6}$$

that derive from the vanishing of the coupling forces for the three particles, where r_* and m_* are the Planck-particle Compton radius (1.62×10^{-33} cm) and mass (2.18×10^{-5} gm) respectively. Equations (5) and (6) are used below to determine the proton overlap radii and lead to the quadrature formula

$$x = 1 + V/m_p c^2 + \ln x \qquad \text{with} \qquad x \equiv r_p/r\,. \tag{7}$$

The proton p-p overlap radius is determined by setting $V = 2m_pc^2$ in (7) and results in

$$x = 3 + \ln x \tag{8}$$

which leads to $x = 4.50$ and the p-p overlap radius $r_2 = r_p/4.50 = 0.047$ fm, where $r_p = 0.210$ fm. The proton core polarizes the resulting vacuum "exposed" by the overlap (see Appendix A).

The negative energy maximum associated with the PV is $-m_ec^2$. Thus the proton e-p overlap radius results from $V = m_pc^2 + m_ec^2$ and yields

$$\begin{aligned} x &= 1 + (m_pc^2 + m_ec^2)/m_pc^2 + \ln x \\ &= 2 + r_p/r_e + \ln x \approx 2 + \ln x \end{aligned} \tag{9}$$

where from (6) $m_e/m_p = r_p/r_e = 1/1836 = 0.00054$. Solving (9) leads to $x = 3.15$ and $r_3 = r_p/3.15 = 0.067$ fm for the e-p overlap radius. (The corresponding Compton wavelength $2\pi r_3$ is 0.42 fm.) The sphere within this radius represents the total "exposed" portion of the PV and the surface of the sphere takes on a positive polarization charge.

The size of the core $(-e_*, m_e)$ in the Dirac electron (whose equation is (3) with r_e replacing r_p) is no larger than $r_e/39,000$ [4] [6, pp. 402-403]; so it is reasonable to conclude that the proton core is similarly reduced in size below r_p.

From the preceding the following picture of the proton structure emerges: the "point charge" proton core has a radius $r_1 < r_p/39,000$; the p-p overlap radius r_2 is 0.047 fm; and the e-p overlap radius r_3 is 0.067 fm; where the proton Compton radius $r_p = 0.210$ fm. The e-p surface at r_3 sustains a polarization charge caused by the core-polarization of the PV.

3 Electron Structure

Calculating the electron structure is similar to the previous section, the quadrature formula being

$$x = 1 + V/m_ec^2 + \ln x \qquad \text{with} \qquad x \equiv r_e/r\,. \tag{10}$$

Setting $V = 2m_ec^2$ yields $x = 4.5$ and $r = r_e/4.5 = 86$ fm for the electron e-e overlap radius, where $r_e = 386$ fm.

To find the electron e-p overlap radius, the potential is set to $V = m_ec^2 + m_pc^2$ and leads to

$$x = 2 + r_e/r_p + \ln x = 1838 + \ln x \tag{11}$$

using $r_e/r_p = 1836$. The solution to (11) is $x = 1846$ and $r = r_e/1846 = (1836/1846)r_p = 0.209$ fm is the overlap radius.

In summary: the electron core radius is less than $r_e/39,000$; the e-p overlap radius is 0.209 fm; and the e-e overlap radius is 86 fm; where the electron Compton radius $r_e = e_*^2/m_ec^2 = 386$ fm.

If the electron were truly a point particle, it would not need to be accelerated to a high velocity to penetrate the proton structure. The effect of velocity on the "electron size" can be seen by examining the electron de Broglie radius.

The relativistic de Broglie radius r_d for the moving electron is derived from the Lorentz invariant constant [7, Appendix A]

$$\frac{e_*^2}{r_e^2} - \frac{m_ec^2}{r_e} = 0 \tag{12}$$

associated with the vanishing of the electron-PV coupling force, and results in

$$r_d = \frac{r_e}{\beta\gamma} = \frac{e_*^2}{\beta E} = \frac{e_*^2/c}{p} = \frac{\hbar}{p} \tag{13}$$

where $\beta = v/c$ (not to be confused with the β in (2) and (3)) and $\gamma = 1/\sqrt{1-\beta^2}$. The relativistic energy and momentum of the electron are $E = m_e\gamma c^2$ and $p = m_e\gamma v$ respectively. The corresponding de Broglie wavelength is $2\pi r_d$.

Using the second ratio in (13), the incident electron energy needed to shrink the "electron radius" to proton levels is

$$E = \frac{e_*^2}{\beta r_d} \approx \frac{e_*^2}{r_d} \tag{14}$$

as β is close to one. For example, (14) shows that to reduce the "electron radius" to the proton Compton radius r_p, $r_d = r_p$ and $E = e_*^2/r_p$.

The energy needed to reduce the de Broglie radius to the proton e-p radius is

$$E = \frac{e_*^2}{r_3} = \frac{3.15 r_e}{r_p}\, m_ec^2 = 2.95\,\text{GeV} \tag{15}$$

where $m_ec^2 = 0.511$ MeV. The corresponding Compton wavelength is $2\pi r_d = 2\pi r_3 = 0.42$ fm.

4 Summary and Comments

The total proton overlap sphere begins at $r_3 = r_p/3.15$, continuing inward through $r_2 = r_p/4.5$, to some radius $r_1 < r_p/39,000$ within which the proton core resides. The core polarizes this overlap sphere and the polarization charge on the spherical surface at $r_3 = 0.067$ fm represents the outer boundary of the apparent spread of the core charge.

However, the electron has structure that obscures the conclusion of the previous paragraph. As an incident

2.95 GeV electron has a de Broglie radius equal to r_3 and can begin to penetrate the proton e-p sphere, the distance of nearest approach for the electron and proton is $2r_3$ or 0.134 fm. The Compton wavelength corresponding to this distance is thus $4\pi r_3$ or 0.84 fm — this is roughly the distance that the two experiments mentioned at the beginning of Section 1 have taken to be the proton spread radius.

In summary, according to the previous calculations, the true radius of the proton charge spread is 0.067 fm with a Compton wavelength of 0.42 fm.

The core-charge polarization of the PV leads to an apparent spread in the proton charge that can be roughly expressed in the proton electric field as

$$E(r) = \frac{e(r)}{r^2} \tag{16}$$

where the spread is

$$e(r) = \begin{cases} e_* & , r < r_1 \\ < e_* & , r_1 < r < r_3 \\ \sim e & , r_3 < r < r_p \\ e = \alpha^{1/2} e_* & , r_p \ll r \end{cases} \tag{17}$$

and α is the fine structure constant. An important characteristic of this result is the rapid drop-off between the core charge e_* and the polarization charge at r_3 which (in conjunction with e_*) is starting to approximate the electronic charge e which obtains for large r. This result explains the similar falloff in the QED spread depicted in Figure 11.6 of [8, p.319].

As in the Dirac theory of the electron [4] [9], the vanishing of the coupling force $F(r_p) = 0$ at the proton Compton radius causes harmonic-oscillator-type oscillations across the r_p-sphere within the PV that are the source of the zitterbewegung oscillations related to the Dirac equation. In the proton case, the resulting frequency is $2c/r_p$, or twice the frequency ($\omega = m_p c^2/\hbar = c/r_p$) associated with the proton mass energy $m_p c^2$. This doubling of the mass energy frequency is due to the vacuum dynamics taking place at $r = r_p$.

Finally, it might be argued that the proton (like the electron) is a stable particle because the separation of the coupling constants e_*^2 and $m_p c^2$ in the coupling force (1) is maintained in the Dirac-equation response (2) to that force.

Appendix A: Overlap Radius

In the relativistic Klein Paradox [10, p. 127], a free electron plane wave propagates in the positive z-direction until it collides with the free-space region II in which the vacuum has been distorted by the step-potential

$$e\phi = \begin{cases} 0 & \text{for } z < 0 \text{ (region I)} \\ V_0 & \text{for } z > 0 \text{ (region II)} \end{cases} \tag{A1}$$

that is externally applied to the half-space $z > 0$. Upon collision with the step, the incident wave excites electron-positron pairs, the electrons and positrons propagating in the negative and positive z-directions respectively. In order for there to be pair excitation, the perturbing potential V_0 must satisfy the inequality

$$V_0 > E + m_e c^2 \tag{A2}$$

where $E = \sqrt{m_e^2 c^4 + c^2 p^2}$ and p are the energy and momentum of the plane wave.

For $V_0 = 0$, the positive energy continuum for an electron in regions I and II increases from $m_e c^2$ in the positive energy direction, while the negative energy vacuum continuum decreases from $-m_e c^2$ in the negative energy direction. When the step-potential is imposed on the $z > 0$ half-space, however, the negative energy continuum in region II is increased as a whole by V_0. The electron positive energy continuum and the vacuum negative energy continuum then overlap in region II. The plane at $z = 0$ is referred to in the present paper as an overlap boundary, and region II as the corresponding overlap region.

In the proton rest frame, the proton core (e_*, m_p) is responsible (via the coupling force (1)) for PV distortion and for "exposing" the negative energy continuum to free space where the core resides. The free-space spherical surfaces where the various positive and negative energy continua begin to overlap are defined as overlap radii. In addition the PV is polarizable [2] as witnessed by the fine structure constant $\alpha = e^2/e_*^2$, where e is the electronic charge and e_* is the massless bare charge in (6). The surface at the e-p overlap radius thus develops a polarization charge due to the polarizing effect of the core charge.

The word-quote ("exposing") in the previous paragraph is meant to emphasize that the "exposed" and polarized PV remains a vacuum state; it does not spill over into the free space of the proton core. The overlap radii then are just imaginary markers in free space where the various overlaps begin.

References

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[6] Milonni P.W. The Quantum Vacuum—an Introduction to Quantum Electrodynamics. Academic Press, New York, 1994.

[7] Daywitt W.C. The Source of the Quantum Vacuum. Progress in Physics, v. 1, 27, 2009. There is a error in Appendix A of this paper: in the first line of the last paragraph "$p = \hbar/r_L$" should read "$m\gamma c = \hbar/r_L$".

[8] Aitchison I.J.R., Hey A.J.G. Gauge Theories in Particle Physics Vol. 1. Taylor& Francis, New York, London, 2003.

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